Abstractness of Continuation Semantics for Asynchronous Concurrency

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Federated Logic Conference 2018
Workshop Domains

Oxford, UK

July 7-8, 2018

Motivation and Aim

- A continuation is a semantic representation of the rest a computation [Stratchey and Wadsworth 1974]
- Traditional continuations can express: non-local exits, coroutines, even multitasking and ADA-like rendez-vous
  - However, the traditional continuations do not work well enough in the presence of concurrency [Hieb, Dybvig and Anderson 1994]

Motivation and Aim

- In [Todoran 2000, Ciobanu and Todoran 2014] we introduced a continuation semantics for concurrency (CSC)
- CSC can express concurrent composition as well as various communication and synchronization mechanisms
  - Intuitively, CSC is a denotational scheduler
- In the CSC approach continuations are application-specific structures of computations
  - Rather than the functions to some answer used in the classic technique of continuations

Motivation and Aim

- In this talk we survey some applications of CSC and we investigate the abstractness of CSC
- We present an optimality criterion specific of continuation semantics that we name weak abstractness which
  - Relaxes the completeness condition
  - Preserves the correctness condition of the classic full abstractness criterion [Milner 1977]

Continuation Semantics for Traditional Concurrent Programming Concepts

CSP-like synchronous communication and asynchronous communication [Todoran 2000]	Warren's Andorra Model [Todoran and Papaspyrou 2000]
CSP Extended with Multiple Channels Communication [Ciobanu and Todoran 2015]	

CSP Extended with Multiple Channels Communication [Ciobanu and Todoran 2015]

$$j ::= c?v \mid j\&j$$
$$a ::= v := e \mid c!e \mid j$$
$$s ::= a \mid y \mid s;s \mid s+s \mid s \parallel s$$

$$c_1!e_1 \parallel \cdots \parallel c_n!e_n \parallel (c_1?v_1\& \cdots c_n?v_n)$$
$$\equiv (v_1, \ldots, v_n := e_1, \ldots, e_n)$$

Warren's Andorra Model [Todoran and Papaspyrou 2000]

$$p ::= (y = x;)^* x$$
$$x ::= g \mid \ll o \gg \mid \langle l \rangle \mid \#\langle l \rangle$$
$$\mid y \mid x \parallel x$$
$$g ::= a \mid \text{fail}$$
$$l ::= \epsilon \mid g?x\ (+g?x)^*$$
$$o ::= \epsilon \mid g : x\ (+g : x)^*$$

$$\ll o \gg \parallel a \parallel \langle a_1?x_1 + a_2?x_2 \rangle \parallel \langle l_1 \rangle \parallel \cdots \parallel \langle l_n \rangle$$

Continuation Passing Semantics for Nature Inspired Formalisms

Membrane Computing
[Ciobanu and Todoran 2017]

$\rho = (D; x), \quad x = o_1 \parallel o_4$

$$
\begin{aligned}
D = \; & \text{membrane } M_0 \, \{ \\
& \quad [o_1, o_4] \Rightarrow o_2 \parallel o_4; \\
& \quad [o_2] \Rightarrow o_3 \parallel \text{new}(M_1, l_1, o_1 \parallel o_5); \\
& \quad [o_2] \Rightarrow o_4; \\
& \quad [o_3] \Rightarrow \text{in}(l_1, o_5) \parallel o_5; \\
& \quad [o_4, o_4, o_5] \Rightarrow o_5; \; \}; \\
& \} \\
& \text{membrane } M_1 \, \{ \\
& \quad [o_1] \Rightarrow o_4 \parallel \text{out}(o_4); \\
& \quad [o_5, o_5] \Rightarrow \delta \; \}. \\
& \}
\end{aligned}
$$

Spiking Neural P-Systems
[Ciobanu and Todoran 2018]

$\rho' = (D', x'),$

$x' = \text{send}(\langle a^{2k-1} \rangle, \{N_1\}) \parallel \text{send}(a, \{N_3\})$

$$
\begin{aligned}
D' = \; & \text{neuron } N_0 \, \{ \, r_\epsilon \mid \{N_1, N_2, N_3\} \, \} \\
& \text{neuron } N_1 \, \{ \, a^+/[a] \rightarrow a; 2 \mid \{N_2\} \, \} \\
& \text{neuron } N_2 \, \{ \, [a^k] \rightarrow a; 1 \mid \{N_3\} \, \} \\
& \text{neuron } N_3 \, \{ \, [a] \rightarrow a; 0 \mid \{N_0\} \, \}
\end{aligned}
$$

On the Abstractness of Continuation Semantics

- The full abstractness condition is in general difficult to establish [Milner 1977]
  - Even more difficult in continuation semantics
- We are not aware of any full abstractness result for a concurrent language designed with continuations
- Continuation-passing semantics for sequential languages are not fully abstract [Cartwright, Curien & Felleisen 1994]

On the Abstractness of Continuation Semantics

- Weak abstractness may be useful when full abstractness is difficult (or impossible) to achieve
- We offer a denotational semantics $[\![\cdot]\!]$ for an asynchronous formalism; we use a domain of continuations

$$\mathbf{D} = \mathbf{Cont} \to \mathbf{P} \qquad\qquad \mathbf{Cont} = \cdots \mathbf{D} \cdots$$

- The semantics is designed by using metric domains [De Bakker and Zucker 1982, America and Rutten 1989]
  - Like the classic domains [Scott 1976, Scott 1982], metric spaces can also be used to express denotational semantics
  - We prove that $[\![\cdot]\!]$ is weakly abstract w.r.t. an $\mathcal{O}[\![\cdot]\!]$

Classic Full Abstractness [Milner 1977]

- A denotational semantics $\mathcal{D} : L \rightarrow \mathbf{D}$ is said to be fully abstract with respect to a (corresponding) operational semantics $\mathcal{O} : L \rightarrow \mathbf{O}$ if
  - $\mathcal{D}$ it is correct with respect to $\mathcal{O}$
    $$\forall x_1, x_2 \in L[\mathcal{D}(x_1) = \mathcal{D}(x_2) \;\Rightarrow\; \forall S[\mathcal{O}(S(x_1)) = \mathcal{O}(S(x_2))]]$$
  - $\mathcal{D}$ and complete with respect to $\mathcal{O}$
    $$\forall x_1, x_2 \in L[\mathcal{D}(x_1) \neq \mathcal{D}(x_2) \;\Rightarrow\; \exists S[\mathcal{O}(S(x_1)) \neq \mathcal{O}(S(x_2))]]$$

(S is an L syntactic context)

Abstractness of Continuation Semantics

- In continuation semantics, $\mathcal{D} : L \to \mathbf{D}$, $\mathbf{D} = \mathbf{Cont} \to \mathbf{F}$, the completeness condition (of full abstractness) is:

$$\forall x_1, x_2 \in L[\,(\exists\gamma \in \mathbf{Cont}[\mathcal{D}(x_1)\gamma \neq \mathcal{D}(x_2)\gamma]) \Rightarrow (\exists S[\mathcal{O}(S(x_1)) \neq \mathcal{O}(S(x_2))])\,]$$

- In continuation semantics for concurrency (CSC) [Todoran 2000, Ciobanu and Todoran 2014] we cannot prove this condition for the whole domain of continuations
  - A weaker condition can be established if we restrict the investigation to the class of denotable continuations
    - Which is closed under arbitrary computations
  - This condition we call weak completeness

The Class of Metric Domains for CSC

- A metric denotational domain of CSC is given as the unique solution of an equation [America and Rutten 1989]

$$\mathbf{D} \cong \mathbf{\Gamma} \stackrel{1}{\to} \mathbf{F}$$

- The class $(\mathbb{\Gamma} \in)\mathcal{DCONT}$ of domains for CSC is:

$$\mathbf{\Gamma} ::= \tfrac{1}{2} \cdot \mathbf{D} \mid \mathbf{M} \to \mathbf{\Gamma} \mid \mathbf{M} \times \mathbf{\Gamma} \mid \mathbf{M} + \mathbf{\Gamma} \mid \mathbf{\Gamma} \times \mathbf{\Gamma} \mid \mathbf{\Gamma} + \mathbf{\Gamma}$$

($\mathbf{M}$ is an arbitrary set $(m \in)M$ endowed with the discrete metric)

Resumptions and Denotable Continuations

- In the CSC approach a continuation is a structured configuration of (partially evaluated) denotations
- Following [De Bakker and De Vink 1996] we use the term resumption as an operational counterpart of the term continuation
- Let $(x \in)X$ be a fixed set. We define a class $(R \in)\mathcal{RES}(X)$ of sets of resumptions for X by:

$$R ::= X \mid M \to R \mid M \times R \mid M + R \mid R \times R \mid R + R$$

(here $(m \in)M$ is an arbitrary set)

Resumptions and Denotable Continuations

- For any language $(x \in)L$ and CSC domain $\mathbf{\Gamma}$ we can construct a corresponding set of resumptions by a homomorphism $res^L(\cdot) : \mathcal{DCONT} \to \mathcal{RES}(L)$

$$
\begin{aligned}
&res^L(\tfrac{1}{2} \cdot \mathbf{D}) = L \\
&res^L(\mathbf{M} \to \mathbf{\Gamma}) = M \to res^L(\mathbf{\Gamma}), \\
&res^L(\mathbf{M} \times \mathbf{\Gamma}) = M \times res^L(\mathbf{\Gamma}) \\
&res^L(\mathbf{M} + \mathbf{\Gamma}) = M + res^L(\mathbf{\Gamma}) \\
&res^L(\mathbf{\Gamma}_1 \times \mathbf{\Gamma}_2) = res^L(\mathbf{\Gamma}_1) \times res^L(\mathbf{\Gamma}_2) \\
&res^L(\mathbf{\Gamma}_1 + \mathbf{\Gamma}_2) = res^L(\mathbf{\Gamma}_1) + res^L(\mathbf{\Gamma}_2)
\end{aligned}
$$

- $res^L(\cdot)$ maps a complete metric space to a plain set
  - $\mathbf{M}$ is a metric space, given by an arbitrary set $(m \in)M$ endowed with the discrete metric

The Class of Denotable Continuations

- Let $(x \in) L$ be a language. Let $\mathcal{D} : L \to \mathbf{D}$, $\mathbf{D} \cong \mathbf{\Gamma} \xrightarrow{1} \mathbf{F}$, be a denotational semantics of L designed with CSC
- $(\gamma \in) \mathbf{\Gamma}^{Cls} = \{ [\![c]\!]^L_{\mathbf{\Gamma}} \mid c \in res^L(\mathbf{\Gamma}) \}$ is the class of denotable continuations for $\mathcal{D}$, where for any $\mathbf{\Gamma} \in \mathcal{DCONT}$, $[\![\cdot]\!]^L_{\mathbf{\Gamma}} : res^L(\mathbf{\Gamma}) \to \mathbf{\Gamma}$ is given by

$$
\begin{array}{rcl}
[\![x]\!]^L_{\frac{1}{2} \cdot \mathbf{D}} & = & \mathcal{D}(x) \\
[\![\lambda m \,.\, c]\!]^L_{\mathbf{M} \to \mathbf{\Gamma}} & = & \lambda m \,.\, [\![c]\!]^L_{\mathbf{\Gamma}} \\
[\![(m, c)]\!]^L_{\mathbf{M} \times \mathbf{\Gamma}} & = & (m, [\![c]\!]^L_{\mathbf{\Gamma}}) \\
[\![(i, c)]\!]^L_{\mathbf{M} + \mathbf{\Gamma}_2} & = & \begin{cases} (1, c) & \text{if} \quad i = 1 \\ (2, [\![c]\!]^L_{\mathbf{\Gamma}_2}) & \text{if} \quad i = 2 \end{cases} \\
[\![(c_1, c_2)]\!]^L_{\mathbf{\Gamma}_1 \times \mathbf{\Gamma}_2} & = & ([\![c_1]\!]^L_{\mathbf{\Gamma}_1}, [\![c_2]\!]^L_{\mathbf{\Gamma}_2}) \\
[\![(i, c)]\!]^L_{\mathbf{\Gamma}_1 + \mathbf{\Gamma}_2} & = & \begin{cases} (1, [\![c]\!]^L_{\mathbf{\Gamma}_1}) & \text{if} \quad i = 1 \\ (2, [\![c]\!]^L_{\mathbf{\Gamma}_2}) & \text{if} \quad i = 2 \end{cases}
\end{array}
$$

The (Metric) Domain of Denotable Continuations

- Let $(\gamma \in)\Gamma^{Dom} = co(\Gamma^{Cls}|\mathbf{\Gamma})$ be the metric completion of Γ^{Cls} with respect to $\mathbf{\Gamma}$ constructed as in the following Remark.
  - We call Γ^{Dom} the domain of denotable continuations for $\mathcal{D}$.

Remark

Let (M, d) be a complete metric space and let X be a subset of M, $X \subseteq M$. We use the notation $co(X|M)$ to represent the set

$$co(X|M) \stackrel{not.}{=} \{x \mid x \in M, x = \lim_i x_i, \forall i \in \mathbb{N} : x_i \in X, \\ (x_i)_i \text{ is a Cauchy sequence in } X\}$$

where the limits are taken with respect to d (as (M, d) is complete $\lim_i x_i \in M$). If we endow $co(X|M)$ with $d_{co(X|M)} = d\restriction co(X|M)$ and X with $d_X = d\restriction X$, then $(co(X|M), d_{co(X|M)})$ is a metric completion of (X, d_X). Recall that each metric space has a completion which is unique up to isometry

The (Metric) Domain of Denotable Continuations

- We have
  - $\Gamma^{Cls} \lhd \Gamma^{Dom}$, and (by construction) $\Gamma^{Dom} \lhd \Gamma$, but
  - in general, $\Gamma^{Dom} \neq \Gamma$

$$((M, d) \lhd (M', d'), \text{ whenever } M \subseteq M' \text{ and } d' \restriction M = d)$$

- In general we do not know whether Γ^{Cls} itself is a complete metric space (where it is, $\Gamma^{Cls} = \Gamma^{Dom}$)

- For a simple asynchronous language, in [Ciobanu and Todoran 2012] it is constructed a continuation $\gamma_\epsilon \in \Gamma$ such that $d(\gamma, \gamma_\epsilon) \geq \frac{1}{2}$, for any $\gamma \in \Gamma^{Dom}$

Weak Abstractness for CSC

Definition

Let $(x \in)L$ be a language and let $\mathcal{D} : L \to \mathbf{D}$, $\mathbf{D} \cong \Gamma \xrightarrow{1} \mathbf{F}$, be a denotational semantics of L designed with CSC. Let also $\mathcal{O} : L \to \mathbf{O}$ be an operational semantics of L and S a typical element of the class of syntactic contexts for L.

(a) *$\mathcal{D}$ is correct with respect to $\mathcal{O}$ iff*

$$\forall x_1, x_2 \in L[\mathcal{D}(x_1) = \mathcal{D}(x_2) \Rightarrow \forall S[\mathcal{O}(S(x_1)) = \mathcal{O}(S(x_2))]]$$

(b) *Let $(\gamma \in)\Gamma^{Dom}$ be the domain of denotable continuations for $\mathcal{D}$. We say that $\mathcal{D}$ is weakly complete with respect to $\mathcal{O}$ iff*

$$\forall x_1, x_2 \in L[(\exists \gamma \in \Gamma^{Dom}[\mathcal{D}(x_1)\gamma \neq \mathcal{D}(x_2)\gamma]) \Rightarrow (\exists S[\mathcal{O}(S(x_1)) \neq \mathcal{O}(S(x_2))])]$$

(c) *We say that $\mathcal{D}$ is weakly abstract with respect to $\mathcal{O}$ iff $\mathcal{D}$ is correct and weakly complete with respect to $\mathcal{O}$.*

Weak Abstractness for CSC

- It suffices to verify the weak completeness property for the class of denotable continuations Γ^{Cls}
  - If weak completeness can be established for Γ^{Cls} then it holds for the whole domain of denotable continuations Γ^{Dom}

Lemma

Let $(x \in)L$ be a language and let $\mathcal{D} : L \to \mathbf{D}$, $\mathbf{D} \cong \Gamma \stackrel{1}{\to} \mathbf{F}$, be a denotational semantics of L designed with CSC. Let also $\mathcal{O} : L \to \mathbf{O}$ be an operational semantics of L and S a typical element of the class of syntactic contexts for L. $\mathcal{D}$ is weakly complete w.r.t. $\mathcal{O}$ iff

$$\forall x_1, x_2 \in L[(\exists \gamma \in \Gamma^{Cls}[\mathcal{D}(x_1)\gamma \neq \mathcal{D}(x_2)\gamma]) \Rightarrow (\exists S[\mathcal{O}(S(x_1)) \neq \mathcal{O}(S(x_2))])]$$

where Γ^{Cls} is the class of denotable continuations for $\mathcal{D}$.

The Language $\mathcal{L}$ - a Paradigm for Asynchronous Communication [Boer, Kok, Palamidessi, Rutten 1993]

Definition

(a) *(Statements)* $x(\in X) ::= a \mid y \mid x + x \mid x; x \mid x \parallel\!\!\lfloor x \mid x \parallel x$

(b) *(Guarded statements)* $g(\in G) ::= a \mid g + g \mid g; x \mid g \parallel\!\!\lfloor x \mid g \parallel g$

(c) *(Declarations, Programs)* $(D \in) Decl = Y \rightarrow G$, $(\rho \in)\mathcal{L} = Decl \times X$

- $(a \in) Act$ is a set of atomic actions ($\delta \in Act$)
- $(y \in) Y$ is a set of recursion variables
- $I : Act \rightarrow \Sigma \rightarrow (\{\uparrow\} \cup \Sigma)$ is an interpretation function
  - If $I(a)(\sigma) = \uparrow$ then a is suspended in σ $(I(\delta)(\sigma) = \uparrow, \forall \sigma \in \Sigma)$

- Instances of the paradigm: asynchronous CCS, asynchronous CSP [Jifeng, Josephs, Hoare 1990], concurrent constraint programming [Saraswat 1993], await statement [Owicki, Gries 1976]

Structure of Continuations (and Resumptions) for $\mathcal{L}$

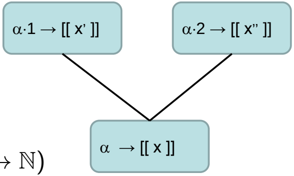
- $(\alpha \in) A = \{1, 2\}^*$ - identifiers (ϵ is the empty sequence)
- $\alpha \leq \alpha'$ iff $\alpha' = \alpha \cdot i_1 \cdots i_n$ $(n \geq 0)$ - partial order

Notation for partially ordered bags (multisets) of computations

$$\{\!|\mathbf{X}|\!\} \quad = \quad \mathcal{P}_{fin}(A) \times (A \to \mathbf{X})$$

- A continuation is a cactus stack (finite tree, active elements at leaves)
- $\nu : (A \times \{\!|\mathbf{X}|\!\}) \to Bool$ $\quad (\nu(\alpha, (\pi, \theta))$ iff α is a leaf in $(\{\alpha\} \cup \pi, \leq_{\{\alpha\} \cup \pi}))$

$$[\![(x' \parallel x''); x]\!] \quad \Rightarrow$$



- Cannot define multisets using $(\mathbf{X} \to \mathbb{N})$

Operational Semantics $\mathcal{O}[\![\cdot]\!]$

Semantic universe, configurations, (consistent) resumptions

$$(p \in)\mathbf{P} = \mathcal{P}_{nco}(\Sigma^* \cup \Sigma^* \cdot \{\delta\} \cup \Sigma^\omega)$$

$$Conf = (X \times CRes' \times \Sigma) \cup (KRes \times \Sigma)$$

$$CRes = A \times KRes, \quad (k \in)KRes = \{\!|X|\!\}, \; k_0 = (\emptyset, \lambda\alpha \,.\, \delta) \in KRes$$

$$CRes' = \{(\alpha, k) \mid \alpha \in A, k \in KRes, \nu(\alpha, k)\}$$

Operational semantics $\mathcal{O}[\![\cdot]\!] : X \to \Sigma \to \mathbf{P}$ $\quad (\mathcal{O} : Conf \to \mathbf{P})$

$$\mathcal{O}[\![x]\!](\sigma) = \mathcal{O}(x, (\alpha_0, k_0), \sigma)$$

$$\mathcal{O}(t) = \begin{cases} \{\Lambda\} & \text{if } t \text{ terminates} \\ \{\delta\} & \text{if } t \text{ blocks} \\ \bigcup\{\sigma \cdot \mathcal{O}(k, \sigma) \mid t \to (k, \sigma)\} & \text{otherwise} \end{cases}$$

Transition System Specification for $\mathcal{L}$

$$(a, (\alpha, k), \sigma) \rightarrow (k, \sigma') \qquad \text{if } I(a)(\sigma) = \sigma'$$

$$(y, (\alpha, k), \sigma) \nearrow (D(y), (\alpha, k), \sigma)$$

$$(x_1 + x_2, (\alpha, k), \sigma) \nearrow (x_1, (\alpha, k), \sigma)$$

$$(x_1 + x_2, (\alpha, k), \sigma) \nearrow (x_2, (\alpha, k), \sigma)$$

$$(x_1 ; x_2, (\alpha, k), \sigma) \nearrow (x_1, (\alpha \cdot 1, [k \mid \alpha \mapsto x_2]), \sigma)$$

$$(x_1 \lfloor\!\lfloor x_2, (\alpha, k), \sigma) \nearrow (x_1, (\alpha \cdot 1, [k \mid \alpha \cdot 2 \mapsto x_2]), \sigma)$$

$$(x_1 \parallel x_2, (\alpha, k), \sigma) \nearrow (x_1, (\alpha \cdot 1, [k \mid \alpha \cdot 2 \mapsto x_2]), \sigma)$$

$$(x_1 \parallel x_2, (\alpha, k), \sigma) \nearrow (x_2, (\alpha \cdot 1, [k \mid \alpha \cdot 2 \mapsto x_1]), \sigma)$$

$$(k, \sigma) \nearrow (k(\alpha), (\alpha, k \setminus \{\alpha\}), \sigma) \qquad \forall \alpha \in max(id(k))$$

CSC Domains and Evaluation Mechanism

CSC domains

$$\begin{array}{rcl} (\varphi \in)\mathbf{D} & \cong & \mathbf{Cont} \stackrel{1}{\to} \Sigma \to \mathbf{P} \\ (\gamma \in)\mathbf{Cont} & = & A \times \mathbf{Kont} \\ (\kappa \in)\mathbf{Kont} & = & \{\!|\, \frac{1}{2} \cdot \mathbf{D} \,|\!\} \end{array}$$

CSC evaluation mechanism

- **Cont** - open continuations (evaluation contexts)
- **Kont** - closed continuations (manipulated by the scheduler)
- ”Evaluate-normalize-schedule” loop

Auxiliary Operators

Nondeterministic choice ($+ : (\mathbf{P} \times \mathbf{P}) \to \mathbf{P}$)

$$p_1 + p_2 = \{q \mid q \in p_1 \cup p_2, q \neq \delta\} \cup \{\delta \mid \delta \in p_1 \cap p_2\}$$

Denotational scheduler

- $kc : \mathbf{Kont} \to \Sigma \to \mathbf{P}$

$$\begin{aligned} kc(\kappa)(\sigma) \;=\; & \text{if } (id(\kappa) = \emptyset) \text{ then } \{\Lambda\} \\ & \text{else } +_{\alpha \in max(id(\kappa))} \kappa(\alpha)(\alpha, \kappa \setminus \{\alpha\})(\sigma) \end{aligned}$$

$[\![x]\!](\alpha, \kappa)(\sigma) =_c p$ is an abbreviation for:

$$[\![x]\!](\alpha, \kappa)(\sigma) = \begin{cases} p & \text{if } \nu(\alpha, \kappa) \\ \{\delta\} & \text{otherwise} \end{cases}$$

Denotational Semantics $[\![\cdot]\!] : X \to \mathbf{D}$ $(\mathcal{D}[\![\cdot]\!] : X \to \Sigma \to \mathbf{P})$

$$
\begin{array}{rcl}
[\![a]\!](\alpha,\kappa)(\sigma) & =_c & \text{if } (I(a)(\sigma) = \uparrow) \text{ then } \{\delta\} \\
 & & \text{else } I(a)(\sigma)\cdot kc(\kappa)(I(a)(\sigma)) \\
[\![y]\!](\alpha,\kappa)(\sigma) & =_c & [\![D(y)]\!](\alpha,\kappa)(\sigma) \\
[\![x_1 + x_2]\!](\alpha,\kappa)(\sigma) & =_c & [\![x_1]\!](\alpha,\kappa)(\sigma) + [\![x_2]\!](\alpha,\kappa)(\sigma) \\
[\![x_1 ; x_2]\!](\alpha,\kappa)(\sigma) & =_c & [\![x_1]\!](\alpha\cdot 1, [\kappa \mid \alpha \mapsto [\![x_2]\!]])(\sigma) \\
[\![x_1 \mathbin{\|\!\!\lfloor} x_2]\!](\alpha,\kappa)(\sigma) & =_c & [\![x_1]\!](\alpha\cdot 1, [\kappa \mid \alpha\cdot 2 \mapsto [\![x_2]\!]])(\sigma) \\
[\![x_1 \parallel x_2]\!](\alpha,\kappa)(\sigma) & =_c & [\![x_1]\!](\alpha\cdot 1, [\kappa \mid \alpha\cdot 2 \mapsto [\![x_2]\!]])(\sigma) + \\
 & & [\![x_2]\!](\alpha\cdot 1, [\kappa \mid \alpha\cdot 2 \mapsto [\![x_1]\!]])(\sigma)
\end{array}
$$

$$
\mathcal{D}[\![x]\!] \quad = \quad [\![x]\!](\alpha_0,\kappa_0), \quad \alpha_0 = \epsilon, \kappa_0 = (\emptyset, \lambda\alpha.[\![\delta]\!])
$$

Isomorphic Resumptions

Definition

Two open resumptions $(\alpha_1, k_1), (\alpha_2, k_2) \in CRes$ *are isomorphic, written* $(\alpha_1, k_1) \cong (\alpha_2, k_2)$*, iff either (1) or (2) is satisfied:*

(1) $\neg\nu(\alpha_1, k_1)$ *and* $\neg\nu(\alpha_2, k_2)$ *(*(α_1, k_1) *and* (α_2, k_2) *are both inconsistent)*

(2) $\nu(\alpha_1, k_1)$ *and* $\nu(\alpha_2, k_2)$ *(*(α_1, k_1) *and* (α_2, k_2) *are both consistent) and there exists a bijection* $\mu : (\{\alpha_1\} \cup id(k_1)) \to (\{\alpha_2\} \cup id(k_2))$ *such that:*

  (i) $\mu(\alpha_1) = \alpha_2$

  (ii) $\mu(\alpha') \leq \mu(\alpha'') \Leftrightarrow \alpha' \leq \alpha'', \quad \forall \alpha', \alpha'' \in (\{\alpha_1\} \cup id(k_1))$

  (iii) $k_2(\mu(\alpha')) = k_1(\alpha'), \quad \forall \alpha' \in id(k_1)$

Class of Denotable Continuations

Definition

We define $[\![\cdot]\!] : KRes \to \mathbf{Kont}$, $[\![k]\!] = (id(k), \lambda\alpha.[\![k(\alpha)]\!])$. *The class of (open) denotable continuations for* $\mathcal{L}$ *is*

$$Cont^{Cls} = \{(\alpha, [\![k]\!]) \mid (\alpha, k) \in CRes\} = A \times Kont^{Cls}$$

where $(\kappa \in)Kont^{Cls} = \{[\![k]\!] \mid k \in KRes\}$.

Syntactic Contexts for $\mathcal{L}$

Definition

The class of syntactic contexts for $\mathcal{L}$ is given by:

$$S ::= (\cdot) \mid a \mid y \mid S; S \mid S + S \mid S \| S \mid S \lfloor\!\lfloor S$$

$S(x)$ is the result of substituting x for all occurrences of $(\cdot)$ in S.

Let $x, \overline{x} \in X$. When $[\![S(x)]\!](\alpha, [\![k]\!]) = [\![S(\overline{x})]\!](\overline{\alpha}, [\![\overline{k}]\!])$ $^{(*)}$ for for all contexts S and for all isomorphic resumptions $(\alpha, k) \cong (\overline{\alpha}, \overline{k})$ $(\in CRes)$, we write:

$$x \simeq \overline{x}$$

$^{(*)}$ implies $\mathcal{D}[\![S(x)]\!] = \mathcal{D}[\![S(\overline{x})]\!]$, for all contexts S

Concurrency Laws in Continuation Semantics [Ciobanu and Todoran 2014]

$$
\begin{array}{rcl}
x_1 + x_2 & \simeq & x_2 + x_1 \\
(x_1 + x_2) + x_3 & \simeq & x_1 + (x_2 + x_3) \\
x + x & \simeq & x \\
(x_1 + x_2); x3 & \simeq & x_1 ; x_3 + x_2 ; x_3 \\
x_1 ; (x_2 + x_3) & \simeq & x_1 ; x_2 + x_1 ; x_3 \\
x_1 ; (x_2 ; x_3) & \simeq & (x_1 ; x_2); x_3 \\
x + \delta & \simeq & x \\
\delta x & \simeq & \delta \\
x_1 \parallel x_2 & \simeq & x_1 \mathbin{\lfloor\!\lfloor} x_2 + x_2 \mathbin{\lfloor\!\lfloor} x_1 \\
a \mathbin{\lfloor\!\lfloor} x & \simeq & a; x \\
a; x_1 \mathbin{\lfloor\!\lfloor} x_2 & \simeq & a; (x_1 \parallel x_2) \\
(x_1 + x_2) \mathbin{\lfloor\!\lfloor} x_3 & \simeq & x_1 \mathbin{\lfloor\!\lfloor} x_3 + x_2 \mathbin{\lfloor\!\lfloor} x_3
\end{array}
$$

Concurrency Laws over the Domain of Denotable Continuations

- We do not know whether the class $Cont^{Cls}$ of denotable continuations is a complete metric space

- However, in CSC computations are (nonexpansive and thus) continuous functions, because

$$\mathbf{D} \cong \mathbf{Cont} \xrightarrow{1} \Sigma \to \mathbf{P}$$

- It is reasonable to study semantic properties in the metric completion of the class $Cont^{Cls}$, i.e., in the domain of denotable continuations

$$Cont^{Dom} = co(Cont^{Cls}|\mathbf{Cont})$$

Concurrency Laws over the Domain of Denotable Continuations

Definition

We say that two open denotable continuations $(\alpha, \kappa), (\overline{\alpha}, \overline{\kappa}) \in Cont^{Dom}$ are isomorphic, and we write $(\alpha, \kappa) \cong (\overline{\alpha}, \overline{\kappa})$, iff there exist sequences $(\alpha_i, k_i)_i, (\overline{\alpha}_i, \overline{k}_i)_i$, (with $(\alpha_i, k_i), (\overline{\alpha}_i, \overline{k}_i) \in CRes, \forall i \in \mathbb{N}$) such that:

- $(\alpha, \kappa) = \lim_i(\alpha_i, [\![k_i]\!]), (\overline{\alpha}, \overline{\kappa}) = \lim_i(\overline{\alpha}_i, [\![\overline{k}_i]\!])$, *and*
- $(\alpha_i, k_i) \cong (\overline{\alpha}_i, \overline{k}_i), \forall i \in \mathbb{N}$.

Concurrency laws hold for the whole domain $Cont^{Dom}$ of denotable continuations

Let $x, \overline{x} \in X$. When $[\![S(x)]\!](\alpha, \kappa) = [\![S(\overline{x})]\!](\overline{\alpha}, \overline{\kappa})$ for all $\mathcal{L}$ syntactic contexts S and for all isomorphic denotable continuations $(\alpha, \kappa) \cong (\overline{\alpha}, \overline{\kappa}) \in Cont^{Dom}$, we write:

$$x \sim \overline{x}$$

Proposition

$x \simeq \overline{x} \Rightarrow x \sim \overline{x}$, for all $x, \overline{x} \in X$.

Remark

*Not all continuations are denotable: $Cont^{Cls} \lhd Cont^{Dom}$ and $Cont^{Dom} \lhd$ **Cont**, but $Cont^{Dom} \neq$ **Cont** [Ciobanu and Todoran 2012]. As a consequence, $[\![\cdot]\!]$ is not fully abstract.*

Weak Abstractness for $\mathcal{L}$

- Recall that $\mathbf{D} \cong \mathbf{Cont} \xrightarrow{1} (\Sigma \to \mathbf{P})$, $\mathbf{Cont} = A \times \{\!|\, \frac{1}{2} \cdot \mathbf{D} \,|\!\}$.
- If we expand the notation $\{\!| \cdot |\!\}$ we get ($\mathbf{Cont} \in \mathcal{DCONT}$):

  $$\mathbf{Cont} = A \times (\mathcal{P}_{fin}(A) \times (A \to \tfrac{1}{2} \cdot \mathbf{D}))$$

  The corresponding set of resumptions is ($CRes \in \mathcal{RES}(X)$)

  $$CRes = res^X(\mathbf{Cont}) = A \times (\mathcal{P}_{fin}(A) \times (A \to X))$$

- The class of denotable continuations is

  $$\begin{aligned} Cont^{Cls} &= \{ [\![(\alpha, k)]\!]^X_{\mathbf{Cont}} \mid (\alpha, k) \in res^X(\mathbf{Cont}) \} \\ &= \{ (\alpha, [\![k]\!]) \mid (\alpha, k) \in CRes \}. \end{aligned}$$

Lemma

For any $x \in X$, $(\alpha, k) \in CRes$ there is an $\mathcal{L}$ syntactic context S such that: $[\![x]\!](\alpha, [\![k]\!]) = \mathcal{D}[\![S(x)]\!] = [\![S(x)]\!](\alpha_0, \kappa_0)$. Furthermore, S does not depend on x, it only depends on (α, k).

Weak Abstractness for $\mathcal{L}$

Theorem

The denotational semantics $[\![\cdot]\!]$ of $\mathcal{L}$ is weakly abstract with respect to the operational semantics $\mathcal{O}[\![\cdot]\!]$.

Proof.

- One can check that $\mathcal{D}[\![x]\!] = \mathcal{O}[\![x]\!], \forall x \in X$, which implies correctness of $[\![\cdot]\!]$ with respect to $\mathcal{O}[\![\cdot]\!]$ [Todoran 2000, Ciobanu and Todoran 2012]
- For weak completeness, suppose $x_1, x_2 \in X$, $(\alpha, k) \in CRes$ are such that $[\![x_1]\!](\alpha, [\![k]\!]) \neq [\![x_2]\!](\alpha, [\![k]\!])$. By previous Lemma $\exists S$ such that
$$\mathcal{D}[\![S(x_1)]\!] = [\![x_1]\!](\alpha, [\![k]\!]) \neq [\![x_2]\!](\alpha, [\![k]\!]) = \mathcal{D}[\![S(x_2)]\!]$$
- Hence, $\mathcal{O}[\![S(x_1)]\!] = \mathcal{D}[\![S(x_1)]\!] \neq \mathcal{D}[\![S(x_2)]\!] = \mathcal{O}[\![S(x_2)]\!]$,
- We conclude that $[\![\cdot]\!]$ is (weakly complete and thus) weakly abstract with respect to $\mathcal{O}[\![\cdot]\!]$.

$\square$

Concluding Remarks and Future Research

- We introduce an optimality criterion specific of continuation semantics that we call weak abstractness
  - Which relaxes the completeness condition of the classic full abstractness criterion [Milner 1977].
- To illustrate the approach we presented a weakly abstract continuation semantics for an asynchronous language
  - We believe similar weak abstractness results can be obtained for various advanced concurrent control concepts
    - Andorra Model [Todoran and Papaspyrou 2000]
    - Multiparty interactions [Ciobanu and Todoran 2015]
    - Nature inspired formalisms [Ciobanu and Todoran 2017]

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